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Time-Varying Currents

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Time-varying currents

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It would be desirable to have a dynamical theory of how ocean current patterns vary with time in response to variation in the patterns of mass, heat and momentum transfer at the surface, but severe difficulties, particularly the uncertain effects of vertical mixing, nonlinear interactions and bottom topography, oppose the development of such a theory, while its evaluation through comparison with observation is impeded by the insufficiency of both input and output data. The characteristic wavenumber components associated with different parts of the input frequency spectrum at different latitudes can, however, be expected to play a particularly important role in determining response. Several of the means by which this may happen are discussed, with some particular reference to the dynamic response of the Indian Ocean to onset of the Southwest Monsoon.

1. General discussion

Ocean currents have been studied by two main methods, which often interact. First and foremost has been the traditional oceanographic method of quantitative observation and description, not only of surface currents but increasingly of data related to current distribution in depth. Secondly, there is the method implicit in this Meeting's title 'Ocean currents and their dynamics': the attempt to relate ocean currents to the meteorological stimuli producing them.

I have personally been active in oceanography for only the past five years, before which I worked mainly in what may be called engineering science. With this background of engineering science, I call the second approach one that asks the questions 'What kind of black box is the ocean?' (or is a particular ocean or part of an ocean); this use of the term black box implying an input–output analysis. Output here signifies the ocean currents and the associated distributions of temperature and salinity, while 'input' means the meteorological stimuli at the air/sea interface: wind stress, heat transfer and the combined effects of evaporation and precipitation.

The use of the term 'black box' here does not imply any restricted view of what type of input–output relationship may be found in the ocean; whether it is nonlinear or linear, stable or unstable, what sorts of time lags in response may be found, etc. It simply implies concentration on the relationships between input and output, whatever they may turn out to be.

A most important aspect of any input–output relationship is steady-state or zero-frequency response. The steady-state response of the oceanic black box is the relation between the time-means of the ocean-current distributions and the time-means of the distributions of wind stress, heat transfer and evaporation/precipitation. The great majority of studies on ocean-current dynamics have concentrated entirely upon elucidating aspects of this steady-state response.

It may be thought possible, however, to make out an even stronger case for studying the dynamics of ocean-current response at non-zero frequencies; in other words, response to transient stimuli. After all, steady-state response in the strict sense is limited in the practical information it can give because there is just one unique long-time average distribution of the meteorological input. To ask how the time-mean of the output, that is, of the ocean-current pattern, is related to that unique mean meteorological input cannot elicit *more* information than simply to ask what *is* the pattern of mean ocean circulation. There can hardly be any practical value in knowing

† Now Sir James Lighthill.

what the mean ocean circulation would have been if the mean atmospheric circulation had been quite different!

We are, to be sure, enormously far away from being able to map the mean ocean circulation itself, by the direct approach of quantitative observation and description. However, it is in principle conceivable that the presently foreseen vast increase in oceanic data gathering, especially at moored data stations, might gradually give us some sort of approximate knowledge of that mean ocean circulation, comparable at least to what now exists for the atmosphere. Whenever that knowledge may be attained, the need for theories on its relation to mean input may become much less; in the meantime, of course, such theories do help to suggest what to measure (particularly regarding deep currents) and where to measure it.

The situation is quite different regarding theories of transient response, that is, of response to variations in the meteorological input about its long-term average. Far from there being a unique pattern of input variation, such patterns of meteorological fluctuation are almost incredibly diverse. Hence a good understanding of response could link very valuably an enormous population of meteorological variations to a comparably large population of ocean-current variations. The latter population, again, could not conceivably be ever known by observation alone. Therefore the dynamics potentially contains a vastly greater amount of information regarding ocean-current variability than could be obtained by any other method whatsoever.

Why are variations in the ocean circulation of interest? Many of the most important reasons for the interest are biogeographical, that is, related to variations in species distributions. Others are navigational, and concerned, for example, with variations in the zones comprising various ice régimes.

Another group of reasons derives from the fact that the black box being studied is a black box with a feedback loop in parallel. The ocean in turn influences the atmosphere. Many authors have observed that, with respect to the variable of temperature alone, this black box is one with positive feedback (as opposed to the loops with negative feedback that are used by engineers for control purposes), and that the feedback takes place with a very considerable lag (according to the old saw 'the ocean is the memory of the atmosphere'). No doubt this element of positive feedback with lag is one of the underlying causes of the long-term variability of weather patterns. These meteorological arguments tending to justify detailed study both of the black box and of its feedback loop are further strengthened when we widen the range of variables discussed beyond temperature to include the ocean currents themselves, since convection is the most important mode of heat transfer within the oceans.

I have tried here to give a consistent line of argument justifying the study of the dynamics of time-varying ocean currents as systems of interest and importance in their own right. The argument usually given, that rectification of the effects of current variations may in turn influence the dynamics of the mean circulation, is merely a secondary reason on this line of argument—a 'bonus' accruing from their study.

Whatever case be made for seeking to determine the character of the ocean-current black box, we must admit that the difficulties of responding to this case with any sort of effective action are formidable. A purely observational approach to the problem seems out of the question, if only because of the impossibility of obtaining enough output data; that is, of gathering ocean-current data with even crudely adequate spread in both space and time. A theoretical approach, founded on the certainty that oceanic response must obey the laws of Newtonian mechanics, is essential, if only to suggest to oceanographers where to look and what to look for.

At present, however, both approaches are faced with a further uncertainty of a rather unexpected kind: a lack of adequate knowledge of the input data. Although the regular systematic gathering of data is in a far more advanced stage in meteorology than in oceanography, there is still a grave shortage of meteorological data over the oceans. Meteorologists are, to be sure, initiating measures to improve this situation, aimed primarily at improving numerical weather forecasting. In the meantime, however, there are still very real difficulties in translating data, however good, on the primary meteorological variables into oceanic input data: particularly, because of the uncertainty regarding coefficients of wind stress and heat transfer.

This problem of deduction of wind stress and heat transfer from wind-velocity data remains exceedingly difficult under oceanic conditions; more difficult by an order of magnitude than under terrestrial conditions. In the latter conditions there is a definite roughness height whose value can be established for different types of terrain, and on which the coefficients depend. Over the ocean the roughness height varies between wide limits as a function of sea state, which in turn depends primarily on the local wind-velocity distribution, but secondarily on the effects of swell excited by winds in quite other part of the ocean. This alone makes intelligible the large scatter in the values of the coefficients that have been inferred from different types of observation.

Under conditions when local sea state is a product of local winds, Wu (1968) correlated the variation of these coefficients fairly well with a Froude number based on the height above the mean sea surface at which the wind velocity is measured. Furthermore, he showed theoretically that his correlation could be expected, using an empirical correlation between wind stress and mean square wave amplitude which expresses the fact that momentum transferred by wind stress goes principally into the waves themselves.

It is important to remember this point which is in conflict with the traditional Ekman view of momentum transfer to the ocean by skin friction. Actually, over any aerodynamically rough surface, most of the wind stress is communicated through pressure drag ('form drag') on roughness elements. In the case of the ocean, this horizontal pressure force cannot excite horizontal currents directly; it can only excite wave trains carrying the transferred momentum. As long as the momentum remains in irrotational motions (that is, remains in, or is transferred between wave modes) there is no significant effect of Coriolis force and no Ekman-layer formation.

It is true that dissipation of wave energy by viscous action must convert the momentum in these surface waves into sheared currents, but such viscous dissipation is a very slow process, in the course of which the momentum may be transported far from where it was originally injected. Admittedly, interaction of waves with turbulence near the sea surface (that is, with an ensemble of random rotational disturbances) would produce the same effect a lot faster. It is, however, possible that the complicated and intense random motions near the sea surface are primarily of an irrotational, wave-like character. For example, theory suggests that waves in a surface region that has become well mixed with respect to density distribution would not generate rotational disturbances through their nonlinear interactions.

These comments are intended to suggest that not nearly enough is known as yet about air–sea interaction phenomena, particularly in those cases where the phenomena are most important (that is, when the sea is roughest and the wind stress greatest). Detailed experiments aimed at establishing much more clearly the nature of this input into the ocean-current 'black box' are urgently awaited, and fortunately some are being carefully prepared at this moment. This work needs to go hand-in-hand with extensive programmes of further gathering of output data on the time-varying ocean currents themselves if we are to make any satisfactory degree of progress towards further elucidation of the response.

Before going on to discuss the contributions that theory can make to help build up an idea of that response, or at least to suggest what observations to attempt, it is necessary to consider one other possible source of information. In the absence of sufficient data on the black box in which we are interested (response of ocean currents to meteorological input), can we fill any gaps in our knowledge from study of a related black box: the response of the ocean tides to tide-raising forces of astronomical origin?

This question views the latter box as distinguishable from the former, realistically because the tide-raising forces are in the form of a line-spectrum with clearly identifiable frequencies; such a line-spectrum is equally identifiable in current-meter records and there is no difficulty except in rather shallow seas in separating that response from the response to meteorological input. Yet the tidal response is still an oceanic response, although to different kinds of forces; and both the volume of data on it, and the rate at which it is increasing, exceed considerably those for response to meteorological input.

Unfortunately, the largest volume of data is on semidiurnal tides, and therefore gives information at a frequency so much higher than the interesting frequency range of the meteorological input that the corresponding responses are quite different in character. Oceans respond to forcing at that frequency in a resonant, gravity-dominated mode, strongly influenced at the higher latitudes by its proximity to the 'inertial frequency' defined by the Coriolis parameter.

These peculiar features do disappear, however, if we concentrate on oceanic response to, for example, the fortnightly or semi-annual components of tide-raising forces. It is probable that a more thorough ocean-wide study of this response would give data of real value for understanding the harder problems of response to meteorological input.

There are still important differences, to be sure, between the two problems: especially with regard to the spatial distribution of the forces. Not only are the tide-raising forces distributed uniformly in depth instead of acting as surface forces, but, still more important, they have very simple distributions in latitude and longitude, which are quite unlike those of the meteorological input. The latter has considerably larger characteristic wavenumbers which we shall see can be expected to alter greatly the type of response. It may also be important that the tide-raising forces are the horizontal gradient of a potential, so that their curl (more strictly the vertical component of their curl) is zero, whereas in oceanic response to wind stress a role of some importance is played by the curl of the wind stress. Nevertheless, a knowledge of oceanic response to spatial distributions of force such as generate the fortnightly and semi-annual tides may well be a clue of some value in indicating aspects of oceanic response to meteorological input in the extreme cases of low wavenumber input.

Passing to the more difficult problem of studying response at non-zero frequencies, or response to transients, in the real ocean-current black box, that is, response to fluctuations in the meteorological input, we can recognize three types of activity of a more or less theoretical nature which can be used to supplement the scanty available volume of quantitative observation, description and comparison of the fluctuations of input and output data. There is analytic theory, mainly linear theory and straightforward modifications thereof (obtained, for example, by going to the next order of perturbation or by inserting boundary layers). There are numerical experiments, in which a high-speed computer is used to evaluate the properties of some mathematical model of an ocean; and for certain purposes there are laboratory experiments in which some aspect of the ocean's behaviour is studied through the use of some laboratory model.

All these methods have grave, but not identical, limitations, from which it seems to follow in the primitive state of our knowledge of time-varying ocean currents that we must seek to extract what we can from all three. This modest appraisal of what each method can do is generally accepted except possibly by some extreme devotees of the method of numerical experiments! It will, however, be suggested in what follows that a rather detailed study of linear theory and its modifications is important for obtaining indications on what types of experiments (usually numerical, but in some cases also laboratory, experiments) can be used to extract which types of information concerning real oceanic response. Linear theory seems, in fact, to be indispensable for building up a framework of ideas within which to view results from other sources.

The same thing has consistently been found in the study of engineering black boxes, which it has similarly proved useful to study by means of linear theory in the first instance. Furthermore, studies of engineering black boxes by linear theory have often been usefully modified in various relatively simple ways to take into account some of the most important effects of nonlinearity; for example, by the use of perturbation techniques or of the method of the describing function.

Of especial importance among results inferred from the linear theory of engineering black boxes is the frequency response of the system. Linear theory gives a useful preliminary picture of frequency response which can then be refined in other ways: by observation, by numerical experiment, or by the concept of an amplitude-dependent frequency response suggested by the method of the describing function. Much the same is true of the ocean-current black box, where frequency response is still of great importance.

In such a case, however, where the quantities of interest are distributed continuously in space as well as in time, there is absolutely equal interest in the wavenumber response. From linear theory, then, we need to obtain a preliminary picture of the combined wavenumber and frequency response of the system: in particular, of which dominant ranges of wavenumber are associated with any given range of frequency. This information is particularly essential in the design either of data-collecting programmes or of numerical experiments, though it may in turn become considerably refined as a result of a really comprehensive body of work of either of these kinds.

Ocean-current response is exceedingly sensitive to the characteristic wavenumbers, as well as to the characteristic frequencies, of the meteorological input. This clear conclusion from linear theory almost certainly remains qualitatively valid in spite of the quantitative inaccuracy of that theory. It implies that any attempt to analyse the action of the input in terms of separate actions by localized components of the input will give quite misleading results. A narrowly localized input (including a δ-function input as the extreme case) possesses a very wide wavenumber spectrum and may accordingly excite a wide variety of responses in the ocean, although when such inputs are combined into a more typical form (with narrower wavenumber spectrum) many of the types of response to individual localized excitations would simply disappear through destructive interference. A good model must therefore attribute the right characteristic range of wavenumbers, as well as of frequencies, to the meteorological input.

I shall attempt to illustrate these points as far as possible in the time available to me. I must emphasize again that I am making only a modest assessment of what linear analysis can do, and making that only in the context of studying response to time-varying inputs. Pedlosky (1969) has made an interesting case along very similar lines to justify having carried out a linear analysis of the steady-state response, but the arguments that follow do not depend on his case being

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accepted. The models discussed are dependent on an assumption that the mean outputs (distributions of current velocity, temperature and salinity) have heen already set up by the action of the mean meteorological inputs, and that we consider time-varying currents as perturbations in those outputs resulting from any perturbations in the inputs. Thus we allow for a pre-existing stratification of temperature and salinity, and in principle we allow also for the fluctuations to be subject to advection by the mean current system. Hitherto, however, there have not yet been many analyses taking such advection into account, and although its effects are mentioned more than once in what follows, most of the actual analysis described neglects it. This means that predicted transient velocity fields are considered as simple additions to any pre-existing mean current distribution.

As one last restriction, the work presented here to exemplify linear analysis and its uses studies the effect of fluctuations in one type of meteorological input only: the wind-stress distribution. The remainder of the lecture is devoted, therefore, to the topic of unsteady wind-driven ocean currents; the subject of my Symons Memorial Lecture (Lighthill 1969*a*) but with the emphasis directed in a generally different manner and primarily towards fundamentals.

2. Barotropic response

The response to wind stress of an ocean with a given pre-existing stratification of temperature and salinity can according to a simplified linear theory (see, for example, Lighthill 1969*b*) be separated into two components: the barotropic component and the baroclinic component. These two components of ocean-current output are predicted as responding in completely different ways to the wind-stress input. This is one of the most important indications of simple theory, and it strongly suggests that observations of time-varying ocean currents can profitably be separated out for purposes of analysis into their barotropic and baroclinic components. Even though nonlinear effects and other effects (such as vertical mixing) neglected in the simplified linear theory must without doubt bring about coupling between the responses of the two components, in ways discussed below, that coupling is expected to be often weak enough for analysis of data into these two components to throw valuable light on the nature of the response.

The barotropic component of response is a response of the ocean as a whole, in which at each location the water at all depths from top to bottom moves with uniform velocity. To separate out the barotropic component involves taking the average of all currents with respect to depth: the current velocity at all depths is, in the barotropic component of motion, equal to this average velocity. The other component of the current distribution, the baroclinic component, is the difference between the true distribution of current velocity and its average with respect to depth.

Thus, all the horizontal momentum in an elementary horizontal area of ocean resides in the barotropic component; the baroclinic component has zero horizontal momentum. Any change of level of the free surface of the ocean is associated exclusively with the barotropic component of motion (through the two-dimensional divergence of this horizontal momentum). By contrast, the baroclinic component of motion should be thought of as a motion in which the free surface remains at a constant level; below the free surface, however, baroclinic motions are associated with tilting of the surfaces of constant density.

In an excellent example of an analysis of time-varying ocean currents into these two components, Phillips (1966) studied a series of observations near Bermuda. He showed that 78 % of the energy in the fluctuations of current was in the barotropic component and 22 % in the baroclinic. This result reminds us how readily the meteorological input is able to excite the ocean into a 'full flow' (the Russian 'polny potok') in which there is participation of water at all depths.

Such excitation might be thought incompatible with the traditional Ekman-layer view of how momentum is transferred from air to ocean in conditions when turbulent diffusion may be important. However, Stommel (1958) showed that there was no incompatibility: in particular, the mechanism since christened 'spin-up' allowed the motions in the Ekman layer, through their horizontal divergence, to excite motions in the ocean as a whole by stretching of vortex lines.

Nevertheless, the effectiveness of excitation of either barotropic or baroclinic motions can be expected to depend very critically, as explained in § 1, on the input's combinations of characteristic frequency and characteristic wavenumber. We shall find in particular that wind stress must act on a sufficiently large horizontal scale to be effective in exciting the barotropic 'polny potok' response.

Although tide-raising forces, being distributed uniformly in depth, excite directly only the barotropic mode, wind stress through its surface action excites all modes of motion. It is a force whose distribution in depth is a δ function concentrated at the surface. The asymmetry of this distribution means that although the barotropic mode (which involves integration of every effect with respect to depth) is excited, the first baroclinic mode is still more strongly excited. If we measure strength of excitation in each mode by the demanded rate of change of surface current (or more accurately of the mean current in the well-mixed layer at the surface), then the excitation of the barotropic mode is typically (Lighthill 1969*b*) only 5 to 10 % of that of the first baroclinic mode, and higher baroclinic modes are excited to a much smaller extent still (of the order of 1 %).

This is an important point but one that can be easily misinterpreted. It must always be considered together with the fact that for particular combinations of input frequency and wavenumber the barotropic mode may respond to a demanded rate of change far more readily than the baroclinic (whose response will be seen in § 3 to be characteristically sluggish under most conditions), to such an extent as easily to outweigh the relatively lower amount of forcing.

The conditions under which components of the meteorological input with frequency ω and with wavenumber k can excite barotropic motions in an ocean of uniform depth are subject to an important law somewhat reminiscent of Heisenberg's uncertainty principle. Provided that both ω and k are measured on a radian scale, so that they are equal to 2π divided by the period and by the wavelength respectively, this principle takes the simple form

$$\omega k < \beta. \tag{1}$$

Here, β is Rossby's beta,

$$\beta = (2\Omega/R)\cos\theta = (2.3\times10^{-11}\cos\theta)\ \mathrm{m^{-1}\,s^{-1}}; \tag{2}$$

namely, the gradient of the Coriolis parameter $f = 2\Omega\sin\theta$ (where Ω = angular velocity of the Earth's rotation and θ = latitude) with respect to northward distance $R\theta$ (where R = Earth's radius).

While the accuracy of particular linearized theories is distinctly questionable, a generalized condition like (1), that has to be satisfied for excitation of barotropic disturbances, may be of more real value than detailed predictions of what disturbances are excited. It requires that inputs with a time-scale ω^{-1} of the order of 1 week have a characteristic extent k^{-1} of at least

100 km. Only an input that 'gets a grip on' the ocean over such a length-scale can excite a barotropic 'full flow' of the ocean beneath it. Furthermore, if the time-scale is halved then the necessary length-scale is doubled.

After the Heisenberg-type principle (1), the next most interesting characteristic predicted by theories of barotropic disturbances refers to the direction of the wavenumber vector $\boldsymbol{k} = (l, m)$; this means the direction (perpendicular to themselves) in which wave crests and other loci of constant phase are moving. For given frequency ω (Longuet-Higgins 1964), the wavenumber vector (l, m) is predicted as lying (see figure 1) on a certain circle of centre $(-\beta/2\omega, 0)$ and of diameter $\beta/2\omega$ (plain line) or slightly less (broken line). The Heisenberg-type condition (1), that $k = \sqrt{(l^2+m^2)}$ must be less than β/ω, follows immediately, but we see also that there is a still

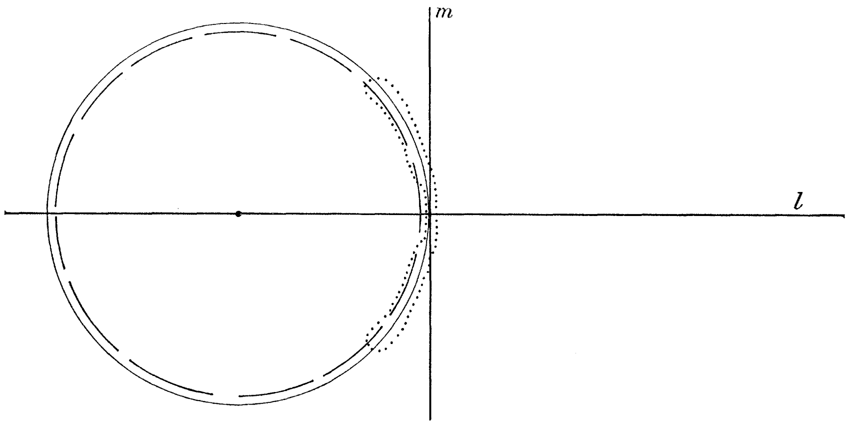

FIGURE 1. Wavenumber diagram for barotropic waves. Both circles have centre $(-\beta/2\omega, 0)$. ——, circle of radius $\beta/2\omega$, given by theory neglecting displacements of the free surface; — —, circle of radius $\sqrt{[(\beta/2\omega)^2-(f^2/gH)]}$, given by more accurate theory; ..., this encloses those parts of the wavenumber diagrams with m considerably greater numerically than l.

stronger condition on disturbances excited by inputs of a nearly zonal character. These must have the wavenumber component m considerably greater numerically than l, whence the wavenumber vector must lie in the region ringed with a dotted line where its magnitude must be considerably less, even, than β/ω.

Figure 1 shows furthermore the direction of the group velocity for waves of each wavenumber, a direction which for each point representing such waves is towards the centre of the circle. The magnitude of the group velocity takes the form βk^{-2} on the simple theory (particularly appropriate to equatorial regions) represented by the plain-line circle, and on a more accurate theory (broken-line circle) is a little less than this. Thus, it is the inputs with smallest wavenumber that excite disturbances with largest group velocity, that is, disturbances whose position moves away from the input region fastest.

Note that inputs of a nearly zonal character produce wave 'packets' or wave 'groups' which as a whole travel in a nearly westward direction with this substantial group velocity, even though the wavecrests are moving quite a lot more slowly in a nearly northward or southward direction (Pedlosky 1965). As far as the barotropic components of ocean currents are concerned, this gives a clearer foundation for understanding the phenomenon of westward intensification of currents, as well as of their fluctuations, than do any explanations based on steady-flow theories.

Since inputs of a nearly zonal character are of particular interest, it is worth considering carefully the physical mechanism underlying this predicted westward travel of the resulting disturbance. Figure 2 shows schematically the author's physical interpretation of this phenomenon on the basis of the simplest theory (plain-line circle in figure 1), which is most accurate in equatorial regions and which he applied to the disturbances generated in the Indian Ocean by the onset of the Southwest Monsoon (Lighthill 1969*b*). The theory is governed by, essentially, a vorticity equation, stating that the 'curl' of the wind stress exerts a local couple that communicates vorticity to the bulk movements of the ocean, by the 'spin-up' mechanism mentioned earlier. This changes the vertical component of the absolute vorticity, which in turn is the sum of a 'relative-vorticity' element due to the ocean currents and a 'planetary-vorticity' element, f, due to the Earth's rotation.

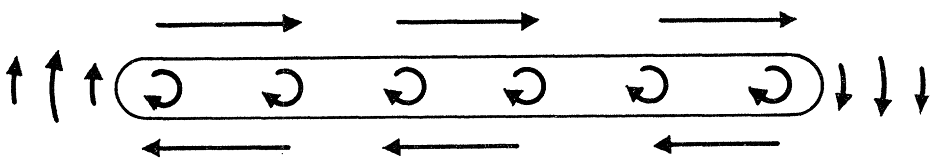

FIGURE 2. Schematic view of the mechanism for predicted westward travel of a barotropic disturbance resulting from a nearly zonal input.

Figure 2 shows schematically a zonal region of finite length, within which negative vorticity has just been generated by onset of a wind-stress pattern whose curl is negative; for example, due to winds with the eastward component of velocity increasing rapidly to the north (such as are observed near the equator during the Southwest Monsoon or in mid-latitudes between the Trades and the Westerlies). This pattern of relative vorticity in the ocean generates eastward flow to the north of it and westward flow to the south of it. The 'return flow' to the west of it is accordingly a northward flow which convects fluid deficient in planetary vorticity into an area just west of the zonal region. By this means, the area of negative relative vorticity spreads continually westward.†

Similar arguments show how this spreading process halts when it encounters a western boundary. The effect of such a boundary on a pattern of negative vorticity can be represented as in figure 3 through an image-vorticity pattern, and as the two patterns come together the region of northward flow gets squeezed between them into an ever-thinning 'western boundary current'. If horizontal momentum exchange is neglected, calculation of this thinning process on linear theory suggests that it is continued indefinitely; a westward-propagating pattern of northward current flux becomes concentrated in a layer whose thickness decreases like $1.4/\beta t$ (with time t measured from when the pattern would have reached the location of the boundary if the ocean had been unbounded); this quantity is reduced to a value of 100 km in about 1 week. The times required are increased by a factor $\sec\alpha$ for a boundary at an angle α to the north–south direction. A more refined theory taking horizontal momentum exchange into account shows the thinning finally brought to a halt when Munk's classical steady-flow value of the boundary-current thickness has been attained.

These results from transient-flow analysis seem to be more clear-cut than results from studying the reflexion of an infinite sinusoidal train of Rossby waves at a boundary, although such studies do predict that waves of arbitrary orientation are 'reflected' as waves with crests parallel to the

† By contrast, southward return flow at the other end of the zonal region ultimately builds up to the steady-state stage, satisfying the 'Sverdrup relation', in that it convects excess planetary vorticity into the region at the same rate as the wind-stress curl removes it.

coast. The synthesis from such individual results of a prediction for reflexion of a wave packet is more cumbersome than the direct transient analysis, whose prediction of the thinning boundary current is, however, well confirmed by numerical studies (Gates 1968, 1969).

These and other numerical studies investigate among other things the influence of nonlinear terms in the equations of barotropic motion, although this is really an illogical procedure: there is no independence of the barotropic and baroclinic parts of the motion if nonlinear effects are significant, and exchange of energy between them due to nonlinear interaction must be at least as important as the effects of self-convection by barotropic disturbances. We may note, however, for what they are worth, the main effects reported from including such self-convection terms in the equations of barotropic disturbances.

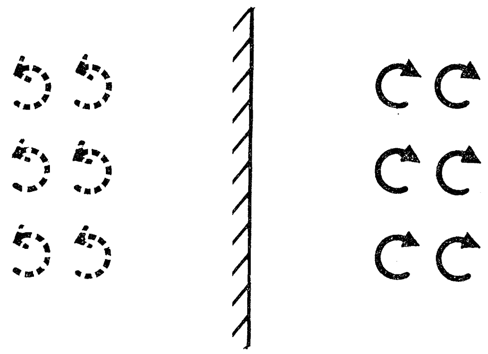

FIGURE 3. Schematic view of the encounter of a westward travelling pattern of negative vorticity with a north–south ocean boundary.

First, the formation of a steady western boundary current without further thinning is facilitated by these nonlinear effects in regions such that the magnitude of the negative wind-stress curl increases polewards. Wherever it decreases polewards, however, the formation of a boundary layer appears to be resisted by nonlinear effects (Kamenkovich 1966). The numerical studies suggest that they may even facilitate separation of the current from a western boundary, as well as some form of 'inertial overshoot' in which the current retains its concentrated jet-like character for a considerably greater distance than is indicated when only the linear effects are considered (Veronis 1966). We remember at this point from Professor Robinson's lecture that nonlinear effects are responsible also for certain types of instability of any separated jet-like flow.

If nonlinear self-convection of the barotropic flow, or even its more complicated nonlinear interactions with baroclinic disturbances, were the only corrections needed to the simple linear theory of barotropic motions in an ocean of uniform depth, the combined use of analytical and numerical techniques might provide such corrections to an adequate degree of approximation. It is much more difficult, however, to be sure about the relevance to real oceans with complex bottom topography of the work that has been described in this section so far; namely, studies of barotropic motion in an ocean of uniform depth. The topography probably modifies the conclusions greatly, except possibly in equatorial regions.

Some clues regarding the modifications in question are suggested by work of Welander (1968*a*, 1968*b*), Rhines (1969*a*, 1969*b*) and others. We may note first the influence of those variations in the ocean depth H which have characteristic wavenumbers considerably smaller than the wavenumber k of the disturbances; that is, topographical variations on a larger scale than that of the disturbances. Both for steady and unsteady flow we can then say that the special properties of the westward direction noted above in the uniform-depth case become transferred to what one may call the 'effectively westward' direction; this is a direction in which f/H remains constant, while increasing to the right of it but decreasing to the left.

This fact is indicated either by precise group-velocity studies, or by a crude vorticity argument of the type exemplified earlier. Only the latter will be given here.

It is convection of planetary vorticity in the 'effectively northward' direction (f/H increasing) that generates negative relative vorticity. This is because convection, such that the depth of ocean in which the convected fluid finds itself increases at the rate DH/Dt, causes the planetary component of that fluid's vorticity, f, to receive enhancement by vortex-line stretching at a rate $(f/H)\,DH/Dt$, while the planetary vorticity of the ambient fluid changes at the rate Df/Dt. The difference in these two rates,

$$\frac{f}{H}\frac{DH}{Dt}-\frac{Df}{Dt}=-H\frac{D}{Dt}\left(\frac{f}{H}\right) \tag{3}$$

gives the resulting rate of change of relative vorticity, which as foretold is negative if and only if f/H is increasing.

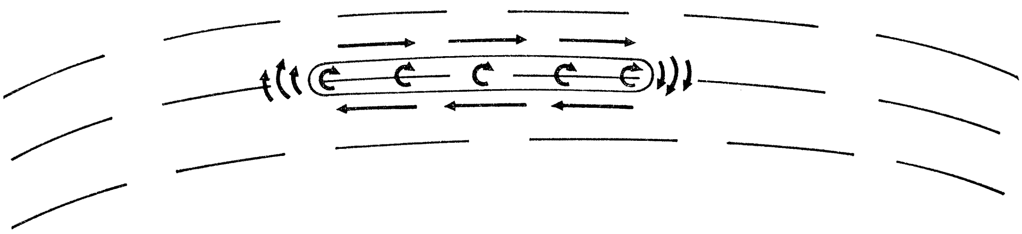

FIGURE 4. Schematic view of the mechanism for 'effectively westward' travel of a barotropic disturbance (that is, travel along the broken-line curves f/H = constant).

It follows that a region of negative vorticity which is extended much more in the 'effectively east–west' direction (that is, along a line f/H = constant) than in the direction at right angles can propagate itself 'effectively westwards' by the mechanism illustrated in figure 4 (which is an analogue or extension of that illustrated in figure 2). The vortex layer generates 'effectively eastward' flow on one side of it and 'effectively westward' flow on the other (effectively southern) side. These motions do not generate vorticity changes. To the 'effective west' of the region there is however a return flow, in an 'effectively northward' direction, that generates negative vorticity and thus propagates the wave. In the meantime (see the earlier footnote) an 'effectively southward' return flow builds up at the back of the wave to a steady state when it is increasing relative vorticity locally at just the rate at which it is being removed by any wind-stress curl.

A recent determination (Gill & Parker 1970) of lines f/H = constant for the world oceans, using smoothed data for H as is appropriate since the above theory applies only to the large-scale depth variations, is shown in figure 5. This indicates how very substantial are the differences between 'effective west' and 'true west' except near the equator. It does suggest, on the other hand, that, even in middle latitudes, most disturbances originating in mid-ocean and propagating along lines f/H = constant should end up on the western boundary as in the constant-depth case.

Some reasonable approximations may be given by such theoretical considerations when topographical variations are on a considerably larger scale than that of the disturbances, but there is much greater uncertainty in other cases. Work such as that of Carrier (1970) suggests that in the diametrically opposite case (small-scale topographical variations) the barotropic disturbance is progressively scattered and thus gradually attenuated with distance. This source of attenuation is additional to any attenuation due to turbulent frictional dissipation. Still

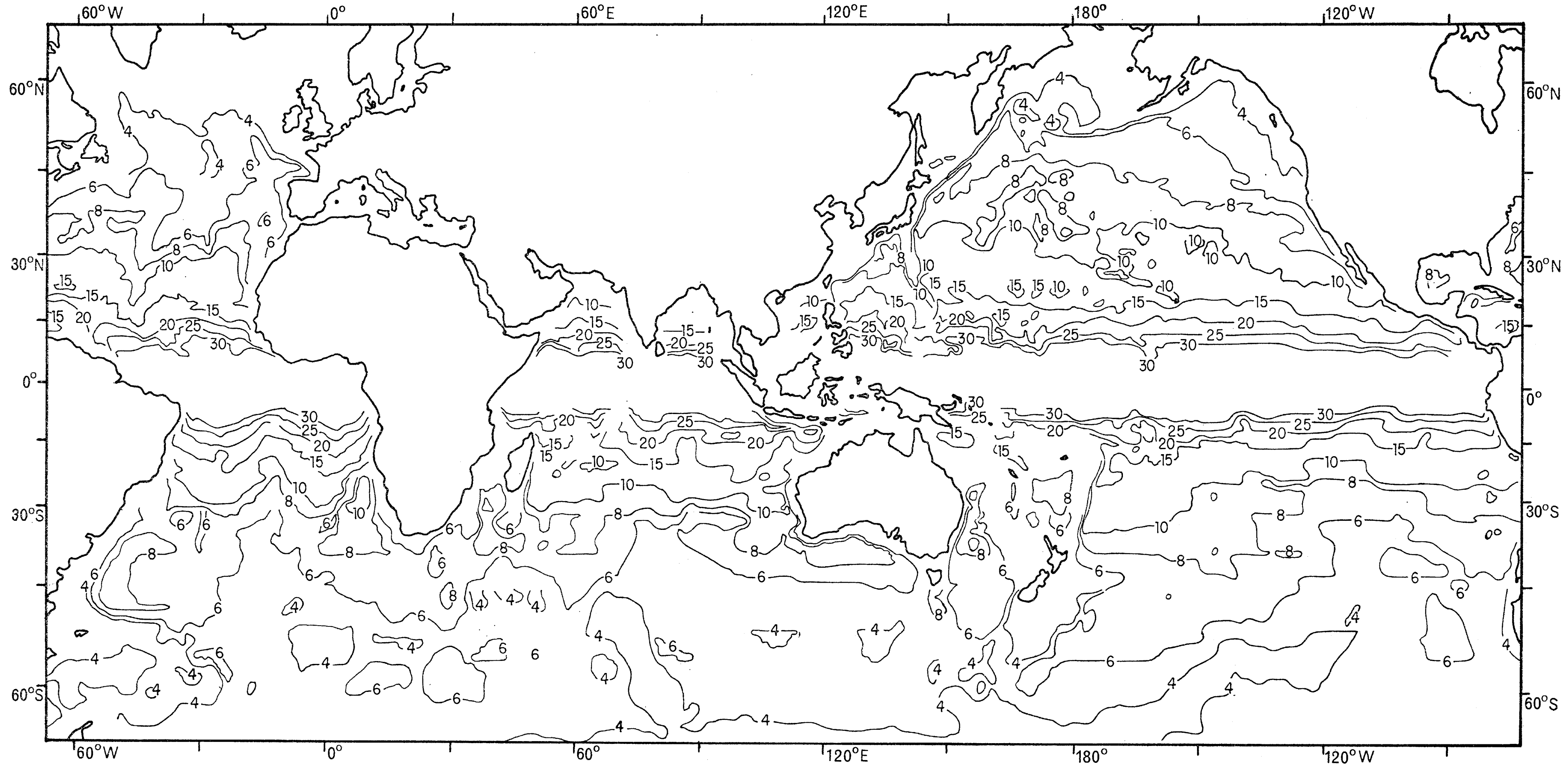

Figure 5. Contours of $D = H \operatorname{cosec} \theta$ for the world oceans, drawn on a mercator projection. Values are given in kilometres. The corresponding value of f/H on each contour is given by $f/H = 2\Omega/D$, where $2\Omega = 1.47 \times 10^{-4}\,\mathrm{s}^{-1}$ is twice the rate of rotation of the Earth. Contours are not drawn where the actual depth H is less than 3 km. From Gill & Parker (1970).

greater interaction (leading to still greater effective attenuation) is probable when the scales of barotropic disturbance and topographical are comparable, and is suggested by work of Rhines. It may also be noted that discontinuous features of bottom topography (ridges and steps on the bed) are expected to make possible various types of trapped wave or edge-wave confined to the neighbourhood of the discontinuity (Rhines 1969*a*).

All these difficulties in the theory of barotropic disturbances are greatly reduced near the equator, where f is small. This means that on the left-hand side of (3) the first term, depending on depth, but containing the f factor, is small compared with the second term (which is equal to β times the northward component of fluid velocity). For this reason the author decided that

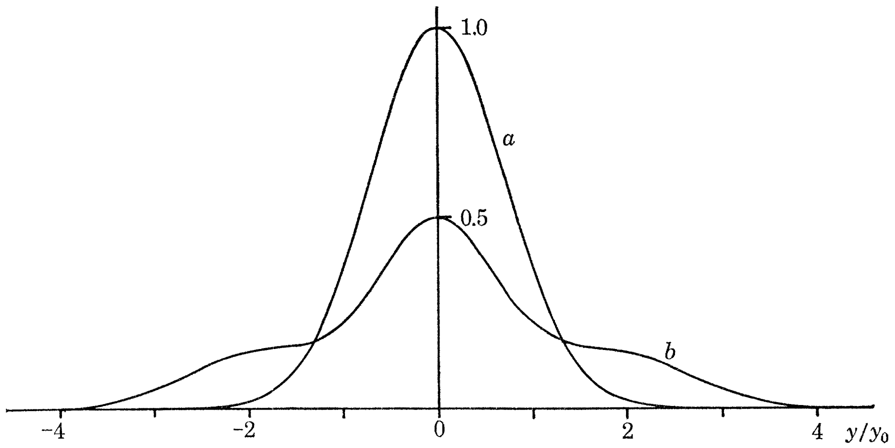

FIGURE 6. Predicted barotropic boundary-current flux (curve *b*) as a function of the northward coordinate y, at such a time after onset of a Gaussian distribution of wind-stress curl (curve *a*) that the peak flux has risen to half its predicted steady-state value.

there was some value in applying barotropic constant-depth disturbance theory to the propagation of the transient currents generated in the Indian Ocean by onset of the Southwest Monsoon. This study, on completion, suggested that there might be a significant barotropic component in the Somali Current (that is, in the resulting western boundary current), contributing as much as a quarter to the surface flow velocity (Lighthill 1969*b*).

A subsidiary suggestion was derived from the expression β/k^2 for the group velocity of current patterns so generated. This implies that those of small wavenumber k will arrive first. Figure 6 indicates how this may spread the disturbance in the north–south direction: a distribution of wind-stress curl with latitude as in curve *a* may generate a distribution of current flux on the western boundary after a relatively short time, when only components of small wavenumber have arrived there, as in curve *b*.

These considerations (regarding detailed modifications of wave patterns as they propagate near the equator) would however be quite out of place in mid-latitude oceans, where it is indeed possible that bottom topography attenuates disturbances excited by meteorological variations almost completely before they can transfer information as far as the western boundary. Recent evidence (Schmitz & Richardson 1968) that the total flux in the Florida Current shows at the very most a variability of 10 % may be regarded as tending to support this view.

Before leaving barotropic disturbances, it is desirable to mention briefly any special characteristics of their excitation by travelling meteorological forcing effects such as cyclones. It is important to realize that the effective frequency associated with each wavenumber vector

$\boldsymbol{k} = (l, m)$ is considerably increased for a travelling pattern of forcing input. This can make it harder for the Heisenberg-type principle (1) to be satisfied.

Consider, for example, a steady meteorological pattern moving with velocity U in a direction that makes an angle α (in the positive sense) with the eastward direction. We may expect that

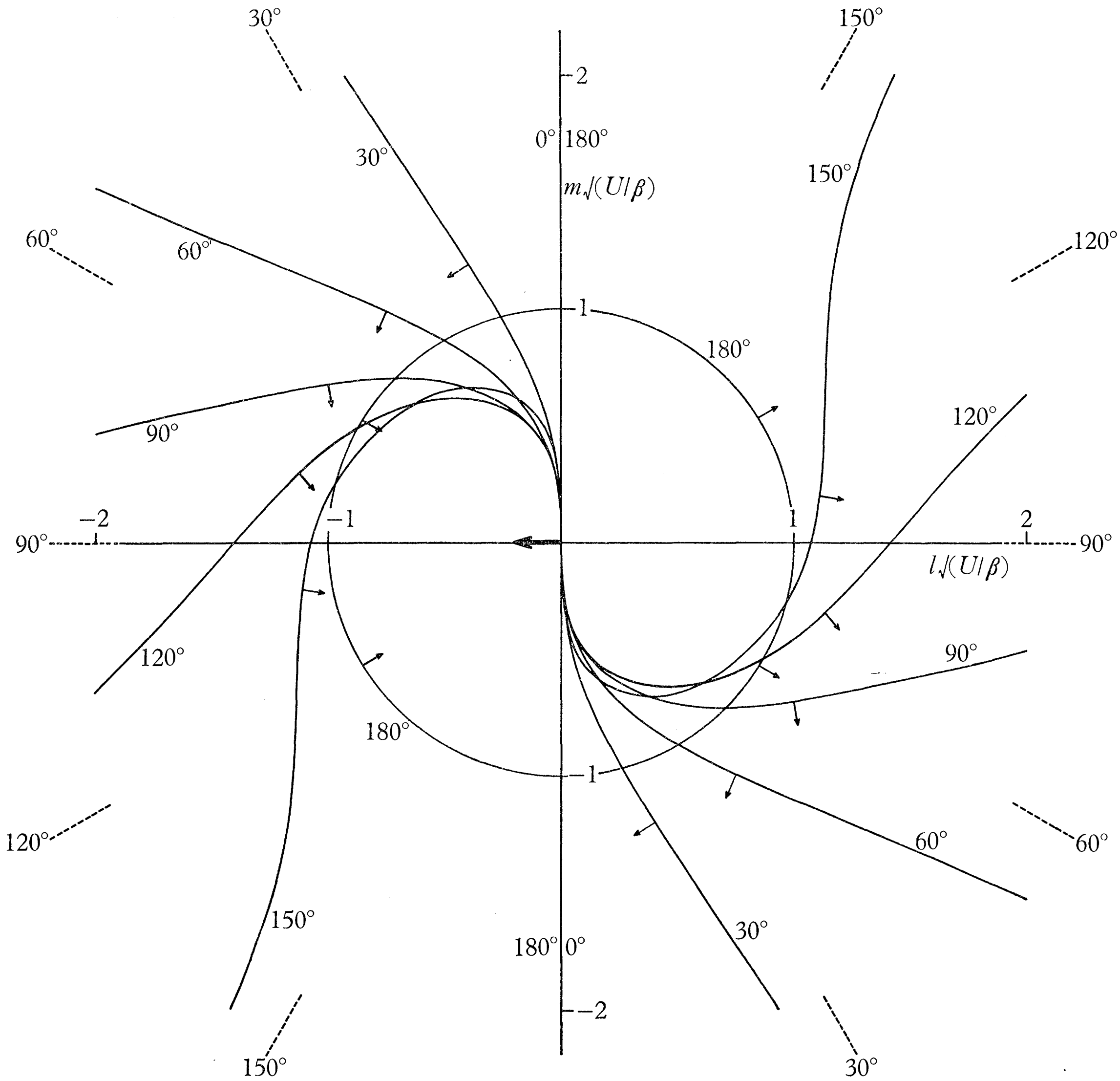

FIGURE 7. Wavenumber diagrams for barotropic disturbances excited in an ocean of uniform depth by a steady meteorological input travelling with uniform velocity U in directions making positive angles $\alpha = 0$, 30, 60, 90, 120, 150 and 180° (marked on the curves) with the eastward direction. Broken lines are asymptotes.

for wavelike components of such a travelling input the frequency is of the order Uk unless the crests are nearly parallel to this direction of travel. Condition (1) then limits k to being less than $\sqrt{(\beta/U)}$ for every barotropic disturbance that may be generated unless it satisfies this special condition.

Detailed calculations bear out this conclusion (Lighthill 1967). The exact equation for ω is

$$\omega = U(l\cos\alpha + m\sin\alpha). \tag{4}$$

Figure 7 shows for different values of α the range of values of (l, m) which satisfy equation (4) while lying on the plain-line circle (with radius $\beta/2\omega$) of figure 1. For values of α other than 0 or 180° there is a marked difference between the predicted behaviour inside and outside the circle $k = \surd(\beta/U)$; outside the circle it tends to satisfy the 'special condition' just noted, which amounts to requiring the term in parentheses in (4) to be small. The arrows on the curves indicate the directions, relative to the centre of the disturbance, in which wave packets with given wave-number (l, m) are predicted to be found. They fill a wedge comprising every direction between the westward direction, characteristic of wavenumbers small compared with $\surd(\beta/U)$, and the direction trailing behind the travelling input, characteristic of wavenumbers large compared with $\surd(\beta/U)$. Here the value of $\surd(\beta/U)$ would, for example, be $1/(600\,\text{km})$ for $U = 6\,\text{m}\,\text{s}^{-1}$. This might mean that the disturbances generated by a large travelling cyclone would include wave-numbers both smaller and larger than this and therefore would fill practically the whole of such a wedge.

The cases $\alpha = 0$ and 180° are special: in the first place any wavenumber that exactly satisfies the 'special condition', here $l = 0$ (characteristic of a zonal pattern of excitation), is capable of exciting barotropic disturbances whatever the value of m; for $\alpha = 0°$ (eastward travel) they trail behind the travelling input, while for $\alpha = 180°$ the components of wavenumber k exceeding $\surd(\beta/U)$ do so but those of greater wavenumber are found to the west (since *their* group velocity βk^{-2} exceeds U). In the second place, for $\alpha = 180°$ any wavenumber $\boldsymbol{k}$ with magnitude k exactly equal to $\surd(\beta/U)$ can excite disturbances, which should be found in the eastward semi-circle (as the arrows on the circle $k = \surd(\beta/U)$ in figure 7 indicate) relative to the travelling input.

In the temperate zones there is particular interest in cases with α relatively small, when the theory for a travelling steady meteorological input predicts disturbances confined to rather a narrow wedge behind it. Study of whether this conclusion remains valid for inputs of finite duration required essentially that frequency analysis be applied to such inputs, and suggested that when most important components of the input have frequencies less than $\surd(U\beta)$ the con-clusion remains valid. Typically this means that cyclones of long duration (a week or more) may generate barotropic disturbances that are of particularly significant amplitude because they are thus confined within a narrow wedge.

These considerations suggest that major travelling cyclonic wind patterns may possibly bring about transient barotropic disturbances that could be one of the influences promoting variable deflexions of jet-like currents (such as the separated Gulf Stream). It would be interesting to attempt an investigation of this source of variability that may possibly exist in addition to the instability mechanisms of which Professor Robinson has reminded us.

3. Baroclinic response

Baroclinic response is inherently more complicated than barotropic response because, if a general distribution of current velocity with depth is analysed into normal modes, just one of those is the barotropic mode (independent of depth), whereas there is an unlimited sequence of different baroclinic modes corresponding to a given density stratification (see, for example, Lighthill 1969*b*). They are called the first, second, third, ..., modes according to the number of 'nodes' or zeros in the associated current distribution. When, however, the stratification of density (more strictly, the vertical distribution of that density which would at atmospheric pressure correspond to the local values of temperature and salinity) takes a form typical of either tropical or mid-latitude oceans, then the first baroclinic mode can be regarded as of over-riding importance.

This first baroclinic mode is associated with tilting of the main thermocline, and includes a corresponding steeply sheared gradient of current. This is combined with current velocities in the opposite direction at great depths, so that the total horizontal momentum is zero. Propagation of the first baroclinic mode is influenced by the Earth's rotation and by gravity acting on the above-mentioned density differences.

Somewhat surprisingly, the propagation of this mode satisfies equations identical with those for barotropic propagation but with greatly reduced values of the product gH which occurs in those equations. This product represents for barotropic disturbances the tendency for the free surface to revert to a flat condition.

For baroclinic disturbances there is a similar tendency for a tilted main thermocline to revert to a flat condition, but because it is generated by gravity acting on quite small differences of density above and below the thermocline the tendency operates far more slowly; this enormously reduced rate can, as linear theory shows, be represented by a reduced effective value of gH. One can if one likes describe the baroclinic propagation as like the barotropic but with a value of g effectively reduced (by at least three orders of magnitude), but the author prefers to give g a constant meaning and to regard the first baroclinic mode as possessing a particular 'effective' value of the depth H. This is typically of the order of 1 m instead of the actual topographical depths of a few kilometres.

An important consequence of this small effective value of H in baroclinic motion is that the correction (f^2/gH), which demanded use of the broken-line rather than the plain-line circle in figure 1 for example, is always significant. Indeed, that correct broken-line circle is real, implying unattenuated propagation of baroclinic disturbances, only if the frequency is less than

$$\tfrac{1}{2}\beta(gH)^{\frac{1}{2}}f^{-1} \simeq (\cot\theta)/(4\times 10^6\,\mathrm{s}). \tag{5}$$

This shows already how sluggish is baroclinic response unless the latitude θ is very small.

In mid-latitude oceans, provided we are interested in meteorological inputs with wavenumbers small compared with

$$f/(gH)^{\frac{1}{2}} = 1/(20\operatorname{cosec}\theta\,\mathrm{km}), \tag{6}$$

as would usually be the case, we are limited to the almost straight part of the broken-line circle near the origin, where l almost remains constant and equal to $-f^2\omega/gH\beta$. This makes both the phase velocity and group velocity westward in direction and of equal magnitude

$$gH\beta/f^2. \tag{7}$$

Veronis & Stommel (1956) first drew attention to this important limiting case of *non-dispersive* baroclinic Rossby waves.

It is above all this very small value (7) of their velocity in mid-latitude oceans (about 1 cm s^{-1}; see figure 9 below) which makes those oceans so exceedingly slow to respond baroclinically. This conclusion, actually, is unaltered if we remove the restriction that the wavenumber of the input be small compared with (6). For example, meteorological inputs of a nearly zonal character and north–south wavenumber m generate disturbances with group velocity nearly westward and of magnitude

$$\frac{\beta}{m^2+(f^2/gH)}, \tag{8}$$

which tends to the value (7) for small m but never exceeds that value. (Note that it equals the characteristic barotropic value β/m^2 only for very large m, large even compared with (6).)

Evidently this maximum propagation speed $gH\beta/f^2$ is of great significance in mid-latitude oceans, where indeed linear theory predicts that current patterns on a scale large compared with 20 cosec θ km are propagated in a simple non-dispersive manner, without change of waveform, at this speed towards the west. Accordingly, I (Lighthill 1969*b*) made a careful analysis of those terms in the equations which bring about this non-dispersive propagation, in order to understand its physical mechanism. My conclusions may be simply described as follows.

For the exceedingly low frequencies (less than (5)) with which we are concerned, rate of change of relative vorticity is negligible compared with the effects of convection and of thermocline raising (or lowering) upon planetary vorticity, and accordingly these last two effects are in balance. A baroclinic motion that is southward above the thermocline (and therefore northward at great depths) would convectively produce increased vorticity above the thermocline (by convection from higher latitudes) and decreased vorticity below it. Both effects are balanced out, however, by a raising of the thermocline which shortens the vortex lines above it and stretches those below. Actually, f times a rate of rise of the thermocline must on this argument balance β times a measure of the north–south current velocities.

The other half of the argument is that if to the west such north–south motions and associated rising of the thermocline have not as yet started, then the thermocline is tilted downwards towards the west and this generates geostrophically in that location a similarly sheared motion (southwards above the thermocline, northwards at great depths) to bring about the westward propagation. In fact, f times a measure of the north–south current velocities so produced equals the angle of tilt of the thermocline times the effective gH for baroclinic motions. Thus, it is these two simple mechanisms, relating current velocities to thermocline rate of rise or angle of tilt with coefficient (f/β) and (gH/f) respectively, that are responsible for the non-dispersive propagation of such a north–south baroclinic motion towards the west at the speed $gH\beta/f^2$, a ratio of the two coefficients.

The distribution of current with depth in the first baroclinic mode may be quite complicated. Figure 8 indicates a calculated distribution in which (taking, for example, southward flow to the right) there is a rapid drop in southward velocity across the main thermocline and then a more gradual drop as the temperature falls gradually below it, with northward velocities appearing only at about one-third of the distance to the ocean bottom, although the total northward momentum and southward momentum are equal.

Several important effects neglected in the simplified linear theory of baroclinic propagation described above may possibly modify the propagation to an important extent. Vertical exchange processes may produce a redistribution of momentum that has the effect of transferring energy to higher baroclinic modes. Topographical variation is expected to have some effect, though considerably less than in the barotropic case because baroclinic motions 'feel the bottom' considerably less (e.g. their velocities near the bottom are far smaller than their surface velocities). Spatial variations in the depth and structure of the thermocline may be more influential in causing the 'effective depth' H for baroclinic motions to vary, although on the simple theoretical arguments just given this would merely cause the wave speed $gH\beta/f^2$ to vary with position in the east–west direction (due to variation of H) as well as in the north–south direction (due to variation of β/f^2). The simple non-dispersive character of the westward propagation should nevertheless be unaltered.

Far more important, however, as a source of error in the theory of propagation of the first baroclinic mode in mid-latitude oceans are the nonlinear effects. The predicted speed of propagation of these baroclinic vorticity patterns is so low that equally large movements of those patterns can be generated through their advection by barotropic motions. Furthermore, self-convection of the baroclinic motions transfers energy both into barotropic motions and into higher baroclinic modes.

The actual value of the westward propagation speed $gH\beta/f^2$, which the theory predicts as a maximum value, attained for disturbances on a relatively large scale, is plotted as a function of latitude in figure 9. This emphasizes that the theory is quantitatively unsatisfactory in mid-latitude oceans: propagation velocities are of order $10^{-2}\,\mathrm{m\,s^{-1}}$, whereas advection velocities of the same order of magnitude or a little more are common in those oceans.

Nevertheless, the investigation does have one important negative conclusion: since neither propagation nor advection can transmit signals faster than a few centimetres per second the response time for an ocean 10^4 km wide must be very great (at least a decade). Veronis & Stommel

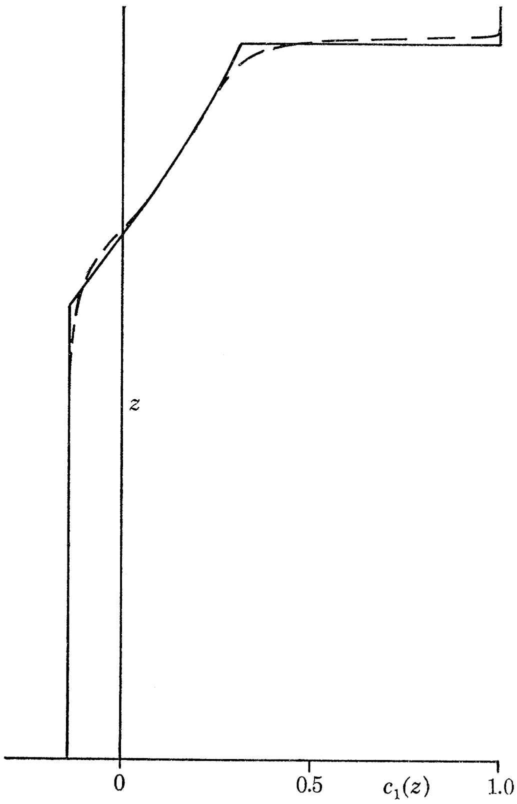

FIGURE 8. Distribution of current velocity $c_1(z)$ with depth z in the first baroclinic mode for density distributions typical of tropical oceans (Lighthill 1969*b*). The broken-line curve is to be preferred.

(1956) first drew this important conclusion about the sluggishness of baroclinic response in mid-latitude oceans. It indicates that changes in meteorological inputs may take at least a decade to modify the baroclinic component of the ocean's current patterns. It was noted earlier that surface stresses give a bigger forcing to the first baroclinic mode than to the barotropic mode, but this is more than counteracted for most practical purposes by the far more sluggish baroclinic response.

Confirmation of the time scales thus predicted is given by the numerical studies of Bryan & Cox (1968), who consider a mid-latitude ocean brought into motion from rest by wind-stress and thermal-input patterns that remain constant from time $t = 0$. A few decades are found necessary (see their figure 5) to bring the ocean-current system close to a steady state. A different set of numerical studies was carried out by Sarkisyan (1966), following methods pioneered by Lineikin (1957), and it is surprising that no similar conclusion emerges. Sarkisyan finds that the displacement of the free surface settles down to a steady state in quite a short time and appears satisfied that the distribution of current in depth does the same. The linear theory indicates however that the response time for free-surface displacement should be a fairly low, barotropic response time, whereas that for current distribution in depth should be an enormously longer, baroclinic response time, and it is difficult to see how any of the effects omitted in the simplified linear theory could fundamentally alter this very great predicted difference.

It is desirable to conclude by a brief account of the major differences in the theory of baroclinic propagation in regions close to the equator (Lighthill 1969*b*). These differences are due to: (i) the very steep rise in the maximum propagation velocity $gH\beta/f^2$ (a rise to above 1 m s^{-1} below 6° latitude as figure 9 shows); (ii) rapid variation in that velocity, leading to a trapping of modes near the equator. At the same time the theory is complicated by two factors: (iii) for all

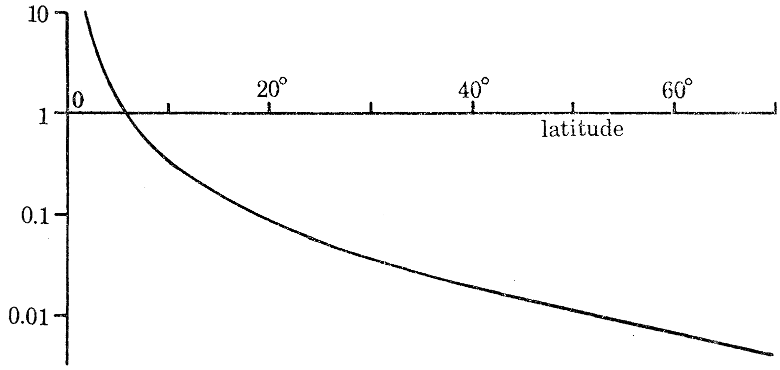

FIGURE 9. Maximum value, $gH\beta/f^2$, of the westward components of group velocity for baroclinic disturbances, plotted in metres per second on a logarithmic scale as a function of latitude, for a value $H = 1$ m typical of the first baroclinic mode.

waves the m^2 term in the denominator of (8) becomes significant near enough to the equator; (iv) this is a region where f takes values close to βy (where y is distance north from the equator) and so the said denominator varies by a large factor across a radian wavelength m^{-1}.

Consideration (iv) shows that simple group-velocity theory cannot be used and we have to use a wave equation with coefficients varying with y, similar to the Schrödinger equation, to calculate these quantized trapped modes. It shows also that the length-scale of these modes in the north–south direction may be expected to be proportional to $(gH/\beta^2)^{\frac{1}{4}}$, a quantity of the order of 200 km. The corresponding time-scale is $(gH\beta^2)^{-\frac{1}{4}}$, a quantity of the order of 1 day.

There is one such eastward-propagating mode, with velocity exactly $(gH)^{\frac{1}{2}}$, and a series of westward-propagating modes with velocities†

$$\frac{(gH)^{\frac{1}{2}}}{2n+1} \quad (n = 1, 2, 3, \ldots). \tag{9}$$

The fastest of these has velocity $\frac{1}{3}(gH)^{\frac{1}{2}}$ which is about 1 m s^{-1}, quite comparable with propagation speeds for barotropic disturbances.

It is these westward-propagating equatorially trapped baroclinic modes that are relevant to the problem of the time-scale of generation of the Somali current on the western boundary of the Indian Ocean in response to the onset of the Southwest Monsoon. The high propagation speeds that are predicted make it possible to imagine that the influence of the wind over a large part of

† Note that these speeds of order $(gH)^{\frac{1}{2}}$ mean that dynamic topography ceases to be reliable as an indication of baroclinic current velocities in the neighbourhood of the equator. Note also that the values (9) are quoted for the case of disturbances on a time-scale significantly larger than $(gH\beta^2)^{-\frac{1}{4}}$.

the ocean may contribute to the setting up of the western boundary current (just as oceanographers have long believed to be the case for, say, the Gulf Stream).

It remains uncertain how far the Somali current is produced by the monsoon winds quite close to the Somali coast and how far by winds blowing over the whole ocean between there and Ceylon. The effect of local winds is indicated by the numerical studies of Cox (1970) to be quite considerable, and in fact comparable with but not numerically so great as the current actually observed. By contrast, I (Lighthill 1969*b*) calculated the effect of the remoter winds, at distances from 500 to 2000 km from the coast, and came to an identical conclusion. Both Cox and I found it easier to infer currents about half of those observed, rather than the full-strength Somali Current. No doubt it is possible that both the local winds and the remoter winds play a significant part in setting it up.

It is possible that nonlinear effects may considerably modify the propagation of the trapped equatorial modes, but such modification may be less than in mid-latitude oceans because the propagation speeds are much higher. An equatorial undercurrent could produce a significant modification, but it should be remembered that onset of the Southwest Monsoon destroys the equatorial undercurrent in the Indian Ocean (Swallow 1967), presumably because it destroys the symmetrical pattern of westward wind stress that maintains it.

The considerations of this section, like those of § 2, suggest that such theories as we possess at present of the response of the oceanic black box described in § 1 are of at most very limited applicability, but may be substantially more useful in equatorial oceans than elsewhere.

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